

Standard Model particles from nonperturbative string theory via spontaneous breaking of Poincare symmetry and supersymmetry

Jun Nishimura^{1,2*} and Asato Tsuchiya^{3†}

¹*KEK Theory Center, High Energy Accelerator Research Organization, Tsukuba 305-0801, Japan*

²*Department of Particle and Nuclear Physics, School of High Energy Accelerator Science, Graduate University for Advanced Studies (SOKENDAI), Tsukuba 305-0801, Japan*

³*Department of Physics, Shizuoka University, 836 Ohya, Suruga-ku, Shizuoka 422-8529, Japan*

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Recently we have shown that (3+1)-dimensional expanding universe appears dynamically and uniquely from the type IIB matrix model, which is a nonperturbative formulation of superstring theory. Here we propose a concrete procedure to identify the local fields corresponding to the massless modes that appear at late times in the same formulation. The spontaneous breaking of (9+1)-dimensional Poincare symmetry and supersymmetry plays a crucial role. We find that the effective field theory below the Planck scale is given generically by a $SU(k)$ grand unified theory with (i) four generations of Weyl fermion with mirror partners and (ii) 21 scalar bosons, both in the adjoint representation of $SU(k)$. We speculate on how the Standard Model emerges at low energy and how the hierarchy problem is solved in this top-down approach.

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Introduction.— The use of a nonperturbative formulation is often indispensable for understanding important dynamical properties of a theory. For instance, confinement of quarks in quantum chromodynamics can be most vividly understood by using the lattice gauge theory [1]. This might also be the case in string theory. Since 1980s, it has been considered that superstring theory has infinitely many vacua that are perturbatively stable. Each of them corresponds to space-time of various dimensionality with various kinds of matter and gauge symmetry. However, it is, of course, possible that the theory has a unique nonperturbative vacuum, which represents a (3+1)-dimensional space-time with the Standard Model (SM) particles propagating on it at low energy.

In order to pursue this possibility, we consider the type IIB matrix model [2], which is a nonperturbative formulation of superstring theory based on type IIB string theory in (9+1) dimensions. This model is particularly suitable for addressing the above issue since the space-time as well as the fields on it is treated as matrix degrees of freedom in a unified fashion. Indeed we have recently found that (3+1)-dimensional expanding universe appears dynamically from the Lorentzian version of the type IIB matrix model [3]. It should be emphasized that the space-time dimensionality seems to be uniquely determined as a result of nonperturbative dynamics. Likewise the SM may emerge uniquely from the matrix model.

In Ref. [3] we have found by Monte Carlo simulation that time as well as space emerges dynamically in the Lorentzian model. Furthermore, we were able to extract the time-evolution, which turned out to be surprising: 9-dimensional space undergoes spontaneous breaking of rotational symmetry at some critical time, after which only three directions start to expand. This phenomenon has been interpreted as the birth of our Universe.

The mechanism of the spontaneous symmetry breaking (SSB) relies crucially on noncommutativity of the space-time represented by 10 bosonic matrices. Therefore, an important issue is whether the usual commutative space-time appears at later times. We addressed this issue by studying the classical equations of motion, which are expected to be valid at late times [4, 5]. There are actually infinitely many solutions representing commutative (3+1)-dimensional space-time. Moreover, we found a simple solution with an expanding behavior, which naturally solves the cosmological constant problem [5]. We consider that there exists a unique solution of this kind that dominates the partition function of the matrix model at late times.

The physically interesting solutions in Ref. [5] break (9+1)-dimensional Poincare symmetry and supersymmetry (SUSY) spontaneously. Therefore, we may naturally assume that the effective field theory well below the Planck scale can be written essentially in terms of the Nambu-Goldstone (NG) modes associated with the SSB and the massless modes obtained by their extension. The former modes decouple from the theory at low energy, but the latter modes do not. In fact it has been considered quite nontrivial to identify the local fields corresponding to the massless modes from the matrix degrees of freedom [6]. We will show that this can be achieved simply by restriction to NG modes and their extension. This is somewhat analogous to the lattice construction of SUSY gauge theory [7], which uses a matrix model with orbifolding conditions. An important difference, however, is that in our case, the orbifolding conditions are not imposed, but similar effects appear dynamically.

SSB in the D-brane background.— As a well-known example in which SSB occurs in string theory, let us first consider a background with D-branes. For concreteness,

we consider type IIB superstring theory with a background of k D3-branes lying on top of each other in ten-dimensional flat Minkowski space. The low energy effective theory on the worldvolume of such D-branes is known to be four-dimensional $\mathcal{N} = 4$ U(k) super Yang-Mills theory, which contains the gauge field, six adjoint scalars and four adjoint Weyl fermions. The existence of the D-branes breaks continuous symmetries, and the associated NG modes can be identified in the low energy effective theory as we discuss below.

First, the translational invariance in six dimensions transverse to the D-branes is broken spontaneously. The associated NG bosons are the U(1) part of six adjoint scalars. Secondly, half of the 32 supersymmetries of type IIB string theory are broken since the D-brane configuration under discussion is half-BPS. The associated NG fermions are the U(1) part of four Weyl fermions.

In fact, the U(k) gauge symmetry appears due to the Chan-Paton factor at the ends of open strings. Hence, each of the above NG modes is enhanced to a set of massless modes which form the adjoint representation of U(k). In this way, the low energy effective theory for the D-brane background is given by the NG modes, their extension to the gauge multiplets and the gauge field.

In addition to the above SSB, the SO(9,1) symmetry is broken down to SO(3,1) \times SO(6). However, the NG bosons associated with this SSB do not appear since even an infinitesimal rotation of the D3-brane induces an infinite change of the field values at space-time infinity.

Matrix model for nonperturbative superstring theory.— Let us next discuss the type IIB matrix model [2], which is a nonperturbative formulation of superstring theory. The action is given by $S = S_b + S_f$ with

$$S_b = -\frac{1}{4g^2} \text{tr} \left([A_M, A_N][A^M, A^N] \right),$$

$$S_f = -\frac{1}{2g^2} \text{tr} \left(\Psi_\alpha (\mathcal{C} \Gamma^M)_{\alpha\beta} [A_M, \Psi_\beta] \right), \quad (1)$$

where A_M ($M = 0, \dots, 9$) and Ψ_α ($\alpha = 1, \dots, 16$) are $N \times N$ Hermitian matrices. The Lorentz indices M and N are contracted using the metric $\eta = \text{diag}(-1, 1, \dots, 1)$. The 16×16 matrices Γ^M are 10D gamma matrices after the Weyl projection, and the unitary matrix \mathcal{C} is the charge conjugation matrix. The action has manifest SO(9,1) symmetry, under which A_M and Ψ_α transform as a vector and a Majorana-Weyl spinor, respectively:

$$\delta_L A_M = \epsilon_{MN} A^N, \quad \delta_L \Psi = \frac{1}{2} \epsilon_{MN} \Gamma^{MN} \Psi. \quad (2)$$

The model also has the fermionic symmetries

$$\delta^{(1)} A_M = i\epsilon_1 \mathcal{C} \Gamma_M \Psi, \quad \delta^{(1)} \Psi = \frac{i}{2} \Gamma^{MN} [A_M, A_N] \epsilon_1 \quad (3)$$

$$\text{and } \delta^{(2)} A_M = 0, \quad \delta^{(2)} \Psi = \epsilon_2 \mathbb{1}_N, \quad (4)$$

as well as the bosonic symmetry

$$\delta_T A_M = c_M \mathbb{1}_N, \quad \delta_T \Psi = 0, \quad (5)$$

where $\mathbb{1}_N$ is the $N \times N$ unit matrix. Let us denote the generators of (3), (4) and (5) by $Q^{(1)}$, $Q^{(2)}$ and P_M , respectively, and define $\tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)}$, $\tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)})$. Then, one obtains the algebra $[\epsilon_1 \mathcal{C} \tilde{Q}^{(i)}, \epsilon_2 \mathcal{C} \tilde{Q}^{(j)}] = -2\delta^{ij} \epsilon_1 \mathcal{C} \Gamma^M \epsilon_2 P_M$ with $i, j = 1, 2$, which is nothing but the ten-dimensional $\mathcal{N} = 2$ SUSY if one identifies P_M with the momentum. (This corresponds to identifying the eigenvalues of A_M with coordinates.) Thus we find that the model possesses the maximal SUSY that type IIB superstring theory has.

NG modes in the type IIB matrix model.— An important feature of the type IIB matrix model is that space-time is treated as a part of dynamical degrees of freedom. It is therefore possible that four-dimensional space-time appears dynamically. The Monte Carlo results of Ref. [3] demonstrate that this really happens. (See Ref. [8] and references therein for earlier works on the Euclidean version of the model.) It implies that continuous symmetries of the model are spontaneously broken, and one obtains NG modes associated with the SSB.

The dominant configurations observed in Monte Carlo simulation [3] show that the elements of A_m ($m = 4, \dots, 9$) are small compared with those of A_μ ($\mu = 0, 1, 2, 3$). Moreover, in the basis which diagonalizes A_0 , the matrices A_i ($i = 1, 2, 3$) have a band diagonal structure, which enables us to extract the time-evolution of the space. Due to the finite size of the matrices, however, we were able to probe only the early time behaviors so far. On the other hand, the classical solutions [5], which are expected to describe the late time behaviors of the matrix model, have tri-diagonal A_μ and $A_m = 0$.

Motivated by these results, we consider a background \hat{A}_M and $\hat{\Psi}$, where $\hat{\Psi} = 0$, and the elements of \hat{A}_m are small in what follows. We also assume that \hat{A}_μ and \hat{A}_m are “close to diagonal”. In this situation, the diagonal elements $(\hat{A}_M)_{ii}$ correspond to the space-time points $(x_i)_M$ in (9+1)-dimensions. By “close to diagonal”, we actually mean that the (i, j) elements corresponding to $\sqrt{(x_i - x_j)^2} \gg \ell$ are small, where ℓ is some length parameter that appears dynamically. (We consider that matrices satisfying this condition represent commutative space-time.) Furthermore, we focus on a sufficiently small space-time region, and assume that the space-time can be approximated by the direct product of (3+1)-dimensional Minkowski space and 6D small extra dimensions. Then the SO(9,1) symmetry is spontaneously broken down to SO(3,1), which naively gives rise to NG bosons corresponding to $\epsilon_{\mu m}$ and ϵ_{mn} in Eq. (2). However, the NG bosons corresponding to the former do not appear since the matrix elements of \hat{A}_μ diverge asymptotically, and so do the elements of $\delta_L \hat{A}_m$. This is analogous to the situation with the D-brane background discussed above. On the other hand, the SSB of translational invariance in six dimensions yields 6 NG bosons corresponding to c_m in Eq. (5). Thus, we obtain 21 NG

bosons in total.

Let us then discuss the SSB of SUSY. Here we assume that ten-dimensional $\mathcal{N} = 2$ SUSY is completely broken. Then, 8 Weyl fermions in four dimensions appear as NG fermions, which correspond to ϵ_1 and ϵ_2 in Eqs. (3) and (4). Indeed, each of ϵ_1 and ϵ_2 , which are Majorana-Weyl fermions in ten dimensions, has 16 real components. In order for the NG fermions corresponding to ϵ_1 to appear, $[\hat{A}_M, \hat{A}_N]$ in the transformation (3) should be asymptotically finite, which we assume in what follows. (In fact, it is nontrivial whether $[\hat{A}_\mu, \hat{A}_\nu]$ and $[\hat{A}_\mu, \hat{A}_m]$ are asymptotically finite since \hat{A}_μ represents an expanding universe. However, we find that this is indeed the case in the physically interesting solutions in Ref. [5].)

Local field theory from the type IIB matrix model.— Below we further assume that the background given by \hat{A}_M and $\tilde{\Psi} = 0$ is a classical solution, and consider the fluctuations \tilde{A}_M and $\tilde{\Psi}$ around the background as

$$A_M = \hat{A}_M + \tilde{A}_M, \quad \Psi = \tilde{\Psi}. \quad (6)$$

We substitute (6) into the action (1). Since the background is a classical solution, there are no terms linear in \tilde{A}_M and $\tilde{\Psi}$.

In order to identify the NG modes, we first consider the following four types of fluctuation:

$$\tilde{A}_\mu = 0, \quad \tilde{A}_m = \phi_{mn} \hat{A}_n, \quad \tilde{\Psi} = 0, \quad (7)$$

$$\tilde{A}_M = 0, \quad \tilde{\Psi} = \frac{i}{2} \Gamma^{MN} [\hat{A}_M, \hat{A}_N] \psi_1, \quad (8)$$

$$\tilde{A}_M = 0, \quad \tilde{\Psi} = \psi_2 \mathbb{1}, \quad (9)$$

$$\tilde{A}_\mu = 0, \quad \tilde{A}_m = \rho_m \mathbb{1}, \quad \tilde{\Psi} = 0, \quad (10)$$

which are associated with (2), (3), (4) and (5), respectively. Here we have introduced ϕ_{mn} , ψ_1 , ψ_2 and ρ_m corresponding to ϵ_{mn} , ϵ_1 , ϵ_2 and c_m , which parametrize the spontaneously broken symmetries in the background. Since the action is invariant under (2), (3), (4) and (5), the quadratic terms with respect to ϕ_{mn} , ψ_1 , ψ_2 and ρ_m do not appear in the action. These are the zero modes associated with the SSB.

Next we identify the low-lying modes associated with the SSB. This is done in field theories by making the parameters in the zero mode fluctuations space-time dependent. In the matrix model, we make the parameters ϕ_{mn} , ψ_1 , ψ_2 and ρ_m depend on the matrix element. For instance, corresponding to (7), we consider

$$(\tilde{A}_\mu)_{ij} = 0, \quad (\tilde{A}_m)_{ij} = (\phi_{mn})_{ij} (\hat{A}_n)_{ij}, \quad (11)$$

and $(\tilde{\Psi})_{ij} = 0$, where $i, j = 1, \dots, N$, and no sum over i and j is taken. Obviously, we may set $(\phi_{mn})_{ij} = 0$ for i and j , which gives $(\hat{A}_n)_{ij} = 0$, namely for $\sqrt{(x_i - x_j)^2} \gg \ell$. When the nonzero components of $(\phi_{mn})_{ij}$ are independent of i and j , (11) reduces to (7) so that the quadratic terms in the action with respect to $(\phi_{mn})_{ij}$ disappear. These facts imply that, for general $(\phi_{mn})_{ij}$, their

quadratic terms coming from $\text{tr}([\hat{A}_M, \tilde{A}_m]^2)$ etc. appear in the form of differences between $(\phi_{mn})_{ij}$ and $(\phi_{mn})_{i'j'}$, where $x_i, x_j, x_{i'}$ and $x_{j'}$ are close to each other. Thus the quadratic terms become local, and they are expected to become the kinetic terms in the continuum limit due to the (3+1)-dimensional Poincare invariance of the background. Obviously, the locality is also guaranteed in the higher order terms, which represent interactions.

In this way, a local field theory for the NG modes is obtained from the matrix model. However, the NG modes interact with each other only through derivative couplings, and they decouple at low energy. Hence it is crucial to introduce the gauge symmetry, which extends the NG modes to a set of massless modes.

Gauge symmetry.— In order to introduce the gauge symmetry in the effective field theory obtained from the matrix model, we consider the background

$$\hat{A}'_M = \hat{A}_M \otimes \mathbb{1}_k, \quad \tilde{\Psi}' = 0, \quad (12)$$

where $\mathbb{1}_k$ is the $k \times k$ unit matrix, \hat{A}_M is the background used in (6), and the matrix model is now considered with the matrix size $N' = Nk$, where the matrix index is represented by a pair of indices (i, a) with $i = 1, \dots, N$ and $a = 1, \dots, k$. The extended background (12) is still a classical solution, and it represents a situation analogous to that of k D3-branes lying on top of each other. We consider that this kind of background is actually realized at late times with some value of k .

In order to discuss the low-lying modes around the background, we consider, for instance, $(\tilde{A}_\mu)_{ia,jb} = 0$, $(\tilde{A}_m)_{ia,jb} = (\phi_{mn})_{ia,jb} (\hat{A}_n)_{ij}$ instead of (11), and assume that $(\phi_{mn})_{ia,jb} = 0$ for (i, j) corresponding to $(\hat{A}_n)_{ij} = 0$. (i.e., $\sqrt{(x_i - x_j)^2} \gg \ell$.) When the nonzero components of $(\phi_{mn})_{ia,jb}$ are independent of i and j , their quadratic terms do not appear in the action. Therefore, these modes are guaranteed to be massless at the tree level.

The background (12) is invariant under a $U(N')$ transformation $\hat{A}'_M \rightarrow U \hat{A}'_M U^\dagger$, where $U_{ia,jb} = \delta_{ij} u_{ab}^{(i)}$ with $u^{(i)} \in U(k)$. This invariance becomes the $U(k)$ gauge symmetry of the effective theory. The $U(k)$ gauge field is expected to appear from $(\tilde{A}_\mu)_{ia,jb}$.

The action for $(\phi_{mn})_{ia,jb}$, $(\psi_1)_{ia,jb}$, $(\psi_2)_{ia,jb}$, $(\rho_m)_{ia,jb}$ and the gauge field has the $U(k)$ gauge symmetry if these massless modes transform as $(\phi_{mn})_{ia,jb} \mapsto u_{ac}^{(i)} (\phi_{mn})_{ic,jd} u_{bd}^{(j)*}$, etc.. (Strictly speaking, the field $(\phi_{mn})_{ia,jb}$ with $i \neq j$ includes the gauge field as well as the adjoint scalar.) Since the $U(1)$ part of the massless modes is decoupled, we obtain a $SU(k)$ GUT with 21 scalars and 8 Weyl fermions in the adjoint representation.

Standard Model.— Let us then speculate on how the SM emerges in the type IIB matrix model at low energy. (See Refs. [9, 10] for earlier works on this issue.) From massless fermionic modes, we actually obtain four generations [11] of Weyl fermion and their mirror partners [12] with opposite chirality. Note that the symmetry between

fermions and their mirror partners is generically broken by a nontrivial structure of \hat{A}_m in the extra dimensions. It is therefore possible that the mirror fermions become heavy and decouple at low energy.

The minimum value of k , for which the GUT can accommodate all the SM particles, is $k = 8$. Let us decompose 8 into $(3, 2, 1, 1, 1)$. Then the 8×8 matrix $(\psi)_{ia,jb}$ for one generation of left-handed Weyl fermion and for fixed i and j has the structure

$$\psi = \begin{pmatrix} \eta_1 & q_L & \eta_2 & u_L & d_L \\ & \eta_3 & (l_R)^c & \eta_4 & \eta_5 \\ & & \eta_6 & \nu_L & e_L \\ & & & \eta_7 & \eta_8 \\ & & & & \eta_9 \end{pmatrix}, \quad (13)$$

where we only write the independent elements explicitly. For instance, the quark doublet appears as q_L , which behaves as the bi-fundamental representation under $SU(3) \times SU(2) \subset SU(8)$. The fields l_R , u_L , d_L , ν_L and e_L are the mirror partners of the corresponding SM fermions. One can easily check that hypercharge can be assigned consistently. The Yukawa interactions arise from $\text{tr}(\Psi C \Gamma^m [A_m, \Psi])$ in (1), whereas the self-interactions of the 21 scalars arise from $\text{tr}([A_m, A_n]^2)$.

Summary and discussions.— In this Letter we proposed a concrete procedure to identify the local fields corresponding to the massless modes that appear at late times in the type IIB matrix model. A crucial role is played by the SSB of Poincare symmetry and SUSY, which was suggested by Monte Carlo results [3]. Assuming that commutative space-time appears at late times, as suggested by classical solutions [5], we find that the locality is guaranteed by the restriction to the NG modes associated with the SSB and their extension.

The effective field theory obtained in this way below the Planck scale has interesting generic features. The gauge group is $SU(k)$ and all the matter fields are in the adjoint representation. This theory with $k = 8$, for instance, can accommodate all the SM particles. (See *e.g.*, Ref. [13] for a work on $SU(8)$ GUT.) We also have the fourth generation [11] and the mirror partners [12], which have been discussed extensively as phenomenologically viable extensions of the SM with various physical motivations. We consider it intriguing that the effective field theory obtained from such a general analysis turns out to be already quite restrictive, and yet the emergence of the SM is not excluded. Whether the SM can really appear from the GUT of this type is an interesting question that can be answered purely within field theory.

The explicit Lagrangian of the GUT can be derived, in principle, from the matrix model once we identify the background \hat{A}_M dominant at late times either by direct Monte Carlo studies or by singling out the classical solution, which is connected smoothly to Monte Carlo results. As we were able to uniquely determine the space-time dimensionality [3], we may be able to obtain the SM

uniquely from this top-down approach.

In our scenario, SUSY is not realized in the particle spectrum, but it still plays the important role of providing the origin of all the fermions below the Planck scale. On the other hand, it remains to be seen how the hierarchy problem is actually solved. One possibility is that, as opposed to our assumption, SUSY is only partially broken by the background above the TeV scale. Otherwise, we can think of various scenarios depending on whether the GUT obtained below the Planck scale is weakly coupled or strongly coupled. In the former case, we may seek for an underlying mechanism for the tree-level conformality [14–16]. In this regard, we point out that the super-renormalizable three-point interactions of the scalars could be extremely small at the tree level since the corresponding terms involve only \hat{A}_m . In the strongly coupled case, we may seek for a scenario with dynamical electroweak symmetry breaking, for instance, by making use of the quarks in the fourth generation [11].

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* Electronic address: jnishi@post.kek.jp

† Electronic address: satsuch@ipc.shizuoka.ac.jp

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